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Semi-leptonic B decays and the two-pion distribution amplitudes

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Abstract. We show that the semi-leptonic decay $B^+ \to \pi^+\pi^-\ell^+\nu_\ell$ can be used as a source of information for two-pion distribution amplitudes. The connection between these amplitudes and the B-meson decay width is achieved by the light cone sum rule method. We show the relevant distribution amplitudes and give the formula for the decay width.

1 Introduction

Recently, two-pion distribution amplitudes (2πDAs) have received considerable interest [1,2] because of their relation to skewed parton distributions [3]. From an experimental point of view the 2πDAs have been mostly discussed for the reaction $\gamma\gamma^* \to \pi\pi$ [4–7], where the factorization has been proven in [8,9], and also in hard exclusive electroproduction [10,11]. Here, we want to add another type of reaction which could provide valuable insight in the dynamics of the formation of two pions, namely semi-leptonic B-decays.

The reaction $B \to \pi\pi\ell\nu$ is an alternative source of information about the 2πDAs as compared to $\gamma\gamma^* \to \pi\pi$ because new structures arise due to the fact that the semi-leptonic weak decay induces an axial vector current in addition to the vector current and the fact that the B-meson is a pseudoscalar particle.

The method used here to connect the 2πDAs with the B-meson decay width is the method of light cone sum rules (LCSR), where the decay amplitude $B \to \pi\pi\ell\nu$ is related to the light cone OPE of the corresponding correlation function. Factorization into the 2πDAs and the hard amplitude here is guaranteed by the large virtuality of the off-shell currents. The light cone sum rule method applied here is essentially the same as used in $B \to \pi\ell\nu$ [12,13] and $B \to \rho\ell\nu$ [14], with the only distinction that the 2πDAs enter. The advance of B-factories may yield a lot of new experimental data on the decay $B \to \pi\pi\ell\nu$, where explicit models for the various 2πDAs entering in this process may be tested and could yield a deepening understanding of the non-perturbative multi-particle dynamics which lies behind these generalized distribution amplitudes.

2 The method

2.1 Kinematics

We consider the process $B^+ \to \pi^+\pi^-\ell^+\nu_\ell$. The kinematics of the process is given by $B(q) \to \ell(p_e) + \nu(q' - p_e) + \pi^+(k_1) + \pi^-(k_2)$ and there exist two light-like vectors n^+ and n^- with $n^+n^- = 1/2$ such that

$$q = (m_B, 0, 0, 0) = m_B \left(n^+ + n^-\right),$$
$$q' = {q'}^- n^+ + {q'}^+ n^- \tag{1}$$

in the rest frame of the B-meson. We can now make a division into "good" (+) and "bad" (-) components, where the "bad" components can be neglected requiring $m_B \gg q'_- > 0$ and $m_B^2 \gg P^2 = (k_1 + k_2)^2 = W^2$. This requirement is necessary to ensure the factorization in the approach of the LCSR technique at the stage where the virtual amplitude is factorized in a hard scattering part and the 2πDAs. The factorization follows in complete analogy with the $\gamma\gamma^*$ case. Under these circumstances $P^2/{m'_B}^2$, using $m'_B = m_B - q'_-$ can be considered as a small expansion parameter and we can decompose:

$$P = q - q' = k_1 + k_2 = m'_B n^+ + \frac{P^2}{m'_B} n^-,$$
$$k_1 = \zeta m'_B n^+ + \bar{\zeta}\frac{P^2}{m'_B} n^- + K_\perp,$$
$$k_2 = \bar{\zeta} m'_B n^+ + \zeta\frac{P^2}{m'_B} n^- - K_\perp, \tag{2}$$

using $\bar{\zeta} = 1 - \zeta$. In this way we have set up a similar light cone decomposition as in the case $\gamma\gamma^* \to \pi\pi$.

For the light cone sum rule technique to apply we need to consider a situation where the B-meson is off shell. One can achieve this in the frame-work of the kinematics discussed so far by simply changing

$$q \to m_B n^+ + \frac{q^2}{m_B} n^-. \tag{3}$$

The two light-like vectors n^+ and n^- are still the same as in (1).

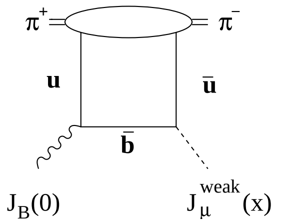

Fig. 1. Diagrammatic representation of the correlation function. The wavy line represents the external "B-meson current" J_B and the dashed line the weak current J^{weak}

2.2 The correlator and the distribution amplitudes

For the application of the LCSR approach one considers a correlation function of a pseudoscalar and a weak current with a quark content that corresponds to the one of a $B^{\pm}$-meson:

$$T_\mu(q,q') = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{-\mathrm{i}q'x}\langle 0|T[J_B(0), J_\mu^{\mathrm{weak}}(x)]|\pi^+\pi^-(P)\rangle,$$
$$J_B(x) = \bar{u}(x)\mathrm{i}\gamma_5 b(x),$$
$$J_\mu^{\mathrm{weak}}(x) = \bar{b}(x)\mathrm{i}\gamma_\mu(1-\gamma_5)u(x). \qquad (4)$$

In this correlation function the B-meson is interpolated by a current J_B with four-momentum q, which is off shell. J_μ^{weak} is the weak $b \to u$ transition current. The diagrammatic representation of T_μ is shown in Fig. 1. Depending on their angular momentum the two pions in the final state can have odd or even parity. With the definition

$$\langle \pi^+\pi^-(P)|\bar{q}(x)\Gamma q(0)|0\rangle_{x^2=0} = \Phi_q^{[\Gamma]}, \qquad (5)$$

we can parameterize the structures that will occur in our correlation function to leading-twist accuracy:

twist $-$ 2 :

$$\Phi_q^{[\mathrm{i}\sigma_{\mu\nu}]} = \frac{(k_{1\mu}k_{2\nu} - k_{2\mu}k_{1\nu})}{2W}\int_0^1 \mathrm{d}z f_{\perp q}^{\pi\pi}(z,\zeta,W^2)\mathrm{e}^{\mathrm{i}zP\cdot x}$$
$$= \frac{(P_\mu R_\nu - R_\mu P_\nu)}{4W}\int_0^1 \mathrm{d}z f_{\perp q}^{\pi\pi}(z,\zeta,W^2)\mathrm{e}^{\mathrm{i}zP\cdot x},$$
$$\Phi_q^{[\gamma_\mu\gamma_5]} = \epsilon_{\mu k_1 k_2 x}\int_0^1 \mathrm{d}z g_q^{\pi\pi}(z,\zeta,W^2)\mathrm{e}^{\mathrm{i}zP\cdot x}$$
$$= \epsilon_{\mu PRx}\int_0^1 \mathrm{d}z g_q^{\pi\pi}(z,\zeta,W^2)\mathrm{e}^{\mathrm{i}zP\cdot x},$$
$$\Phi_q^{[\gamma_\mu]} = P_\mu\int_0^1 \mathrm{d}z f_q^{\pi\pi}(z,\zeta,W^2)\mathrm{e}^{\mathrm{i}zP\cdot x},$$

twist $-$ 3 :

$$\Phi_q^{[\mathbf{1}]} = W\int_0^1 \mathrm{d}z e_q^{\pi\pi}(z,\zeta,W^2)\mathrm{e}^{\mathrm{i}zP\cdot x},$$
$$\Phi_q^{[\gamma_5]} = 0, \qquad (6)$$

using $R_\mu = k_{2\mu} - k_{1\mu}$. In particular, we find $P\cdot R = 0$ and $R^2 = 4m_\pi^2 - W^2$. We only need one of the two Lorentz structures $\mathrm{i}\sigma^{\mu\nu}\gamma_5$ and $\sigma^{\mu\nu}$, as they are related to each other by

$$\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}\gamma_5. \qquad (7)$$

With this decomposition we obtain for the correlation function (4)

$$T_\mu = \mathrm{i}\epsilon_{\mu PRq'}$$
$$\times V_{ub}\left[\frac{2m_b g_u^{\pi\pi}(z,\zeta,W^2)}{((Pz+q')^2 - m_b^2)^2} + \frac{f_{u\perp}^{\pi\pi}(z,\zeta,W^2)/(4W)}{(Pz+q')^2 - m_b^2}\right]$$
$$+\frac{V_{ub}}{(P_z+q') - m_b^2}$$
$$\times\Bigg[m_b P_\mu f_u^{\pi\pi}(z,\zeta,W^2) - (Pz+q')_\mu W e_u^{\pi\pi}(z,\zeta,W^2)$$
$$-\frac{P_\mu R\cdot q' - R_\mu P\cdot(Pz+q')}{4W}f_{u\perp}^{\pi\pi}(z,\zeta,W^2)\Bigg]. \qquad (8)$$

2.3 Calculation of the decay amplitude in the LCSR approach

The next step is to relate the decay amplitude to the distribution amplitudes described to far. For this purpose one inserts a complete set of states with B-meson quantum numbers between the currents [12,13]:

$$T_\mu(q^2) = \int \mathrm{d}^4x \mathrm{e}^{-\mathrm{i}q'x}\frac{\langle 0|\bar{u}(0)\mathrm{i}\gamma_5 b(0)|B\rangle}{q^2 - m_B^2}$$
$$\times\langle B|\bar{b}(x)\mathrm{i}\gamma_\mu(1-\gamma_5)u(x)|\pi\pi\rangle + \ldots$$
$$= \frac{f_B m_B^2}{m_b(q^2 - m_B^2)}\mathcal{M}_\mu + \ldots \qquad (9)$$

using

$$\mathcal{M}_\mu = \int \mathrm{d}^4x \mathrm{e}^{-\mathrm{i}q'x}\langle B|\bar{b}(x)\gamma_\mu(1-\gamma_5)u(x)|\pi\pi\rangle. \qquad (10)$$

The ellipsis indicates all the other hadronic states which in the end we will suppress by Borel transformation. The next step is to define the discontinuity Disc:

$$\mathrm{Disc}\left[T_\mu(q^2)\right] = \frac{1}{2\pi\mathrm{i}}\left(T_\mu(q^2 - \mathrm{i}\epsilon) - T_\mu(q^2 + \mathrm{i}\epsilon)\right), \qquad (11)$$

and isolate $\mathcal{M}_\mu$ using the standard duality approximation,

$$\int_{m_b^2}^{s_0}\mathrm{d}s\mathrm{Disc}\left[T_\mu(s)\right]\mathrm{e}^{-(s-m_B^2)/M^2}\frac{m_b}{m_B^2 f_B} = \mathcal{M}_\mu. \qquad (12)$$

Now one can expand $\mathcal{M}_\mu$ into an orthogonal system,

$$\mathcal{M}_\mu = \int \mathrm{d}^4x \mathrm{e}^{-\mathrm{i}q'x}\langle B|\bar{b}(x)\gamma_\mu(1-\gamma_5)u(x)|\pi\pi\rangle,$$
$$\mathcal{M}_\mu = M_1 P_\mu + M_2 R_\mu + M_3 q''_\mu + \frac{\mathrm{i}}{2W^2}M_4\epsilon_{\mu PRq''},$$
$$q''_\mu = q'_\mu - \frac{R\cdot q'}{R^2}R_\mu - \frac{P\cdot q'}{P^2}P_\mu. \qquad (13)$$

With this decomposition one gets the following LCSR results:

$$M_1 = \int_0^1 \frac{\mathrm{d}z}{z}\frac{\mathrm{e}^{-(s-m_B^2)/M^2}m_b}{m_B^2 f_B}$$

$$\times\left[m_b f_u^{\pi\pi}(z) - \frac{(R\cdot q')(s)}{4W} f_{u\perp}^{\pi\pi}(z) - \frac{1}{2W}\left(s+(2z-1)W^2-{q'}^2\right) e_u^{\pi\pi}(z)\right]\Theta[c(z,s_0^B)],$$

$$M_2 = \int_0^1 \frac{\mathrm{d}z}{z}\frac{\mathrm{e}^{-(s-m_B^2)/M^2} m_b}{m_B^2 f_B}\left[-\frac{(R\cdot q')(s)}{R^2} e_u^{\pi\pi}(z) + \frac{1}{8W}\left(s+(2z-1)W^2-{q'}^2\right) f_{u\perp}^{\pi\pi}(z)\right]\Theta[c(z,s_0^B)],$$

$$M_3 = -\int_0^1 \frac{\mathrm{d}z}{z}\frac{\mathrm{e}^{-(s-m_B^2)/M^2} m_b}{m_B^2 f_B} W e_u^{\pi\pi}(z)\Theta[c(z,s_0^B)],$$

$$M_4 = \int_0^1 \frac{\mathrm{d}z}{z}\frac{\mathrm{e}^{-(s-m_B^2)/M^2} m_b}{m_B^2 f_B}\left[\frac{W}{2} f_{u\perp}^{\pi\pi}(z)\Theta[c(z,s_0^B)] - 4m_b W^2 g_u^{\pi\pi}(z)\left(\frac{1}{zM^2}\Theta[c(z,s_0^B)] + \delta[c(z,s_0^B)] - \delta[-c(z,m_b^2)]\right)\right], \tag{14}$$

using

$$2(R\cdot q')(s) = (\bar\zeta-\zeta)\left(m_B {q'}_+ - \frac{s{q'}_-}{m_B}\right),$$
$$s = \frac{1}{z}\left[z\bar z W^2 + m_b^2 - \bar z {q'}^2\right],$$
$$c(z,s_0^B) = z s_0^B - m_b^2 + \bar z {q'}^2 - z\bar z W^2. \tag{15}$$

Θ and δ functions arise from the continuum subtraction [14]. More precisely we use

$$\frac{-1}{\pi}\mathrm{Im}\int_0^1 \mathrm{d}z\int_{m_b^2}^{s_0^B}\frac{\mathrm{d}s'\mathrm{e}^{-(s'-m_B^2)/M^2}f(z)}{(Pz+q')^2-m_b^2+\mathrm{i}\epsilon}$$
$$= \int_0^1\frac{\mathrm{d}z}{z}\mathrm{e}^{-(s-m_B^2)/M^2}f(z)\Theta[c(z,s_0^B)]\Theta[-c(z,m_b^2)] \quad \text{and}$$
$$\frac{-1}{\pi}\mathrm{Im}\int_0^1 \mathrm{d}z\int_{m_b^2}^{s_0^B}\frac{\mathrm{d}s'\mathrm{e}^{-(s'-m_B^2)/M^2}f(z)}{[(Pz+q')^2-m_b^2+\mathrm{i}\epsilon]^2}$$
$$= -\frac{\mathrm{d}}{\mathrm{d}\alpha}\bigg|_{\alpha=0}\frac{-1}{\pi}\mathrm{Im}\int_0^1 \mathrm{d}z\int_{m_b^2}^{s_0^B}\frac{\mathrm{d}s'\mathrm{e}^{-(s'-m_B^2)/M^2}f(z)}{(Pz+q')^2-m_b^2+\mathrm{i}\epsilon+\alpha}$$
$$= -\int_0^1\frac{\mathrm{d}z}{z}\mathrm{e}^{-(s-m_B^2)/M^2}f(z) \times\left[\frac{1}{zM^2}\Theta[c(z,s_0^B)] + \delta[c(z,s_0^B)] - \delta[-c(z,m_b^2)]\right]. \tag{16}$$

Note that $\Theta[-c(z,m_b^2)] = \Theta[1-z]$, and $\delta[-c(z,m_b^2)] = \delta(1-z)/(m_b^2+zW^2-{q'}^2)$. In order to obtain the square of the decay amplitude, we have to multiply $\mathcal{M}_\mu\mathcal{M}_{\mu'}$ with the leptonic scattering tensor

$$L_{\mu\mu'} = \sum_{ss'}\bar u_s(p_e)\gamma_\mu(1-\gamma_5)u_{s'}(p_\nu) \times\bar u_{s'}(p_\nu)\gamma_{\mu'}(1-\gamma_5)u_s(p_e)$$
$$= 8\Big[(q'_\mu - p_{e\mu})p_{e\mu'} + (q'_{\mu'}-p_{e\mu'})p_{e\mu} - g_{\mu\mu'}(p_e\cdot q' - m_e^2) + \mathrm{i}\epsilon_{\mu\mu' q' p_e}\Big]. \tag{17}$$

Then one gets for the matrix element

$$|M|^2 = \frac{L^{\mu\mu'}\mathcal{M}_\mu\mathcal{M}^*_{\mu'}}{(M_\mathrm{W}^2-q'^2)^2}$$
$$= \Bigg[16(M_1 P\cdot p_e + M_2 R\cdot p_e + M_3 q''\cdot p_e) \times(M_1 P\cdot q' + M_2 R\cdot q' + M_3 q''\cdot q') - 16(M_1 P\cdot p_e + M_2 R\cdot p_e + M_3 q''\cdot p_e)^2$$
$$+4M_4^2\frac{R^2}{W^2}q''^2\left(m_e^2 - \frac{(R\cdot p_e)^2}{R^2} - \frac{(P\cdot p_e)^2}{W^2} - \frac{(q''\cdot p_e)^2}{q''^2}\right) - 8\left(M_1^2P^2 + M_2^2R^2 + M_3^2 q''^2 - M_4^2\frac{R^2}{4W^2}q''^2\right)(p_e\cdot q' - m_e^2) - 8\frac{M_4}{W^2}\epsilon_{\mu\mu' q' p_e}$$
$$\times\epsilon_{\mu P R q'}\left(M_1 P_{\mu'} + M_2 R_{\mu'} + M_3 q''_{\mu'}\right)\Bigg]. \times\frac{1}{(M_W^2-q'^2)^2}. \tag{18}$$

Here we have dropped the weak coupling $(g/(2(2^{1/2})))^4$ for simplicity. We will add it later to the phase-space element in (23). The full expression (18) is rather complicated. To simplify it we make first use of the fact that $P^2/{m'_B}^2$ is small, so that we can throw away all "bad" components connected with n^- which corresponds to an expansion in $P^2/{m'_B}^2$. Then we get approximately the following expressions:

$$\begin{aligned}
2P\cdot q' &= m'_B {q'}^+ + \frac{P^2}{m'_B}{q'}^- \to m_B {q'}^+,\\
2p_e\cdot q' &= (q'^2 + m_e^2),\\
2P\cdot p_e &= m'_B p_e^+ + \frac{P^2}{m'_B}p_e^- \to m_B p_e^+,\\
2R\cdot q' &= (\bar\zeta-\zeta)\left(m'_B {q'}^+ - \frac{P^2}{m'_B}{q'}^-\right)\\
&\to (1-2\zeta)m_B {q'}^+,\\
2R\cdot p_e &= (\bar\zeta-\zeta)\left(m'_B {p_e}^+ - \frac{P^2}{m'_B}{p_e}^-\right) + 4\mathbf{K}_\perp\cdot\mathbf{p_{e\perp}}\\
&\to (1-2\zeta)m_B {p_e}^+ + 4\mathbf{K}_\perp\cdot\mathbf{p_{e\perp}}.
\end{aligned} \tag{19}$$

Here we have defined $p_e^+ = 2p_e\cdot n^+$. The independent variables are now ${q'}^+$, p_e^+ and $\mathbf{p}_{e\perp}$. and one should note that

$${\mathbf{K}_\perp}^2 = \zeta\bar\zeta W^2 - m_\pi^2. \tag{20}$$

We can define the dimensionless quantities

$$x = \frac{{q'}^+}{m_B}, \quad y = \frac{p_e^+}{{q'}^+}. \tag{21}$$

As $m_B \gg q'^-$ we find also

$$\frac{q'^2}{m_B^2} \ll x. \qquad (22)$$

To simplify the expression (18) further, we consider the limit where q' becomes quasi-light-like, i.e. $q'^2 = 0$. Then the leptonic part of the decay process simply factorizes in analogy to the Weizsäcker–Williams approximation in photoproduction; see, e.g., [15].

In this limit q' is quasi-collinear to p_e, and one obtains after integration over the angle related to $\mathbf{p}_{e\perp}$ and after dropping the electron masses for the total decay amplitude

$$\begin{aligned}
&\frac{\mathrm{d}\Gamma_B}{\mathrm{d}\zeta \mathrm{d}x \mathrm{d}y \mathrm{d}q'^2 \mathrm{d}p_{e\perp}^2} \\
&= \frac{G_\mathrm{F}^2 |V_{ub}|^2}{8(4\pi)^5 m_B y^2 x} \frac{\left(1 - \frac{q'^2}{m_B^2 x^2}\right)}{\left(1 + \frac{p_{e\perp}^2}{m_B^2 x^2 y^2}\right)} \Bigg\{ 4m_B^4 x^2 y(1-y) \\
&\times \Bigg[M_1 + (1-2\zeta) M_2 - M_3 \frac{m_B^2}{2W^2} x \\
&\times \left(1 + (1-2\zeta)^2 \frac{W^2}{4m_\pi^2 - W^2}\right)\Bigg]^2 \\
&-32 \left(M_2 - M_3 x \frac{(1-2\zeta) m_B^2}{4m_\pi - W^2}\right)^2 |\mathbf{K}_\perp|^2 |\mathbf{p}_{e\perp}|^2 \\
&+2M_4^2 \frac{m_B^4 x^2}{W^4} |\mathbf{K}_\perp|^2 |\mathbf{p}_{e\perp}|^2 \Bigg\},
\end{aligned}$$

using

$$\frac{G_F}{\sqrt{2}} = \frac{4\pi\alpha_\mathrm{em}}{8\sin^2(\theta_\mathrm{W}) M_W^2}. \qquad (23)$$

3 Discussion

Equation (23) is a simplified form of the decay width in the limits $q'^2 \to 0$ and $P^2/m'^2_B \to 0$. In Appendix A we will show the full result for finite q'^2 and P^2. Of the four distribution amplitudes $f^{\pi\pi}$, $g^{\pi\pi}$, $e^{\pi\pi}$ and $f_\perp^{\pi\pi}$ only the asymptotic form of $f^{\pi\pi}$ is known and there have been attempts to model this function and $f_\perp^{\pi\pi}$ in terms of the instanton vacuum [2].

A calculation of the other distribution amplitudes is beyond the scope of this article. However, we can see if we are able at least to obtain the order of magnitude correct if we neglect all contributions except $f^{\pi\pi}$ where we posses the expression for the asymptotic form and compare the semi-leptonic decay $B \to \pi\pi\ell\nu$ with the semi-leptonic decay $B \to \rho\ell\nu$. Neglecting $e^{\pi\pi}$, $h^{\pi\pi}$ and $g^{\pi\pi}$ may not be so unreasonable, as the first one is twist-3 (i.e. a higher-twist contribution) and the other two are connected to "polarization" states where we know from the experiences of spin physics that the contribution of the quarks is relatively small. If, in this sense, we only retain the contribution from $f^{\pi\pi}$, we obtain a simple expression for the branching ratio $B(B \to \pi\pi\ell\nu)$:

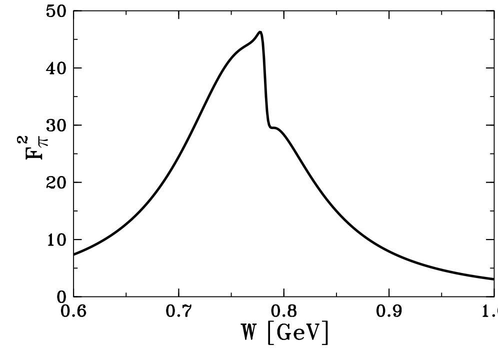

Fig. 2. Square of the time-like pion form factor (F_π^2) in the HLS parameterization

$$\frac{\mathrm{d}B(B \to \pi\pi\ell\nu)}{\mathrm{d}\zeta \mathrm{d}x \mathrm{d}y \mathrm{d}q'^2 \mathrm{d}p_{e\perp}^2} = \frac{G_\mathrm{F}^2 |V_{ub}|^2}{2(4\pi)^5 m_B \Gamma_B y^2 x} \frac{\left(1 - \frac{q'^2}{m_B^2 x^2}\right)}{\left(1 + \frac{p_{e\perp}^2}{m_B^2 x^2 y^2}\right)} \times m_B^4 x^2 y(1-y) \left(M_1^{\mathrm{twist-2}}\right)^2,$$

$$M_1^{\mathrm{twist-2}} = \int_0^1 \frac{\mathrm{d}z}{z} \mathrm{e}^{-(s-m_B^2)/M^2} \Theta[c(z, s_0^B)] \frac{m_b^2}{m_B^2 f_B} f_u^{\pi\pi}(z), \qquad (24)$$

where Γ_B is the total B-meson decay width.

4 Numerical estimates

Now, we try to make some order of magnitude estimates for a comparison between the semi-leptonic decay of a B-meson into two pions on the one hand and into a ρ-meson on the other hand. For $f^{\pi\pi}$ one can use the asymptotic form given in [2]:

$$f_u^{\pi\pi\mathrm{as}}(z, \zeta, W^2) = 6z(1-z)(2\zeta - 1) F_\pi(W^2). \qquad (25)$$

$F_\pi(W^2)$ is the pion form factor in the time-like region, normalized by $F_\pi(0) = 1$. For the pion form factor in the time-like region we use the fit from the CMD2-Collaboration [16] using the hidden local symmetry (HLS) parameterization, which is displayed in Fig. 2. The shape of the time-like pion form factor is a characteristic superposition of the ω and ρ resonances.

For the numerics we take $G_\mathrm{F} = 1.16639 \times 10^{-5} \mathrm{GeV}^{-2}$, $\Gamma_B^{-1} = 1.62 \times 10^{-12}$s, $m_B = 5.279$GeV, and $V_\mathrm{ub} = 0.0035$ [17]. The value of V_ub is an average value where the error assigned to it is of the order of 50%. For the decay width f_B we make use of the corresponding sum rule expression, see e.g. [18]:

$$f_B^2 = \frac{m_b^2}{m_B^4} \exp\left(\frac{m_B^2 - m_b^2}{M^2}\right)$$

$$\times \Bigg[- m_b \langle \bar{q}q\rangle_{\mu^2=M^2} - \frac{m_b}{2M^2}\left(1-\frac{m_b^2}{2M^2}\right)\langle \bar{q}\sigma gGq\rangle_{\mu^2=M^2} + \frac{3}{8\pi^2}\int_{m_b^2}^{s_0^B} s\mathrm{d}s \mathrm{e}^{-(s-m_b^2)/M^2}\left(1-\frac{m_b^2}{s}\right)^2\Bigg], \quad (26)$$

$$\begin{aligned}
\langle \bar{q}q\rangle_{\mu^2} &= \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-4/\beta_0}\langle \bar{q}q\rangle_{\mu_0^2}, \\
\langle \bar{q}\sigma gGq\rangle_{\mu^2} &= \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{2/(3\beta_0)}\langle \bar{q}\sigma gGq\rangle_{\mu_0^2}, \\
\beta_0 &= \frac{11}{3}N_c - \frac{2}{3}n_f, \\
\langle \bar{q}q\rangle_{1\,\mathrm{GeV}^2} &= -245\,\mathrm{MeV}^3, \\
\langle \bar{q}\sigma gGq\rangle_{1\,\mathrm{GeV}^2} &= 0.65\,\mathrm{GeV}^2\langle \bar{q}q\rangle_{1\,\mathrm{GeV}^2}. \quad (27)
\end{aligned}$$

The values of the condensates at $\mu_0 = 1\,\mathrm{GeV}$ have been taken from [14]. In the formula for f_B radiative corrections are not taken into account because they are not taken into account in all the other LCSR calculations presented or used here either. For the same reason we use for α_s the one-loop expression

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}, \quad (28)$$

using $\Lambda^{(5)} = 0.208\,\mathrm{GeV}$ and for the threshold masses in the evolution $m_c = 1.25\,\mathrm{GeV}$ and $m_b = 4.2\,\mathrm{GeV}$ [17]. The sum rule parameters are chosen as in [14]. They can be obtained from Table 1. For the calculation of the decay width the central values have been used, and for the error the maximal deviation to the left and right border. For the dependence on the Borel parameter in Fig. 3 we consider the differential branching ratio at $q'^2 = 0$ using the central values for the other sum rule parameters and integrating $p_{e\perp}^2 \in [0, m_B^2 x^2/4]$, $y \in [0,1]$, $\zeta \in [0,1]$. The value for x is fixed at the ρ pole in the HLS parameterization, i.e. $x = 1 - m_{\rho,\mathrm{HLS}}^2/m_B^2$, with $m_{\rho,\mathrm{HLS}} = 774.57\,\mathrm{MeV}$. This means $x \approx 0.97847$, so effectively x is close to 1 if the invariant mass of the two pions is in the vicinity of the ρ-meson pole. It can be seen that the dependence on the Borel parameter is rather strong, as we only consider the asymptotic form without any higher-twist contributions or radiative corrections. Furthermore, for simplicity, we have kept the other sum rule parameters fixed. In principle they should vary with the Borel parameter as given e.g. in Table 1.

As the ρ-meson decays nearly exclusively into two pions there should be a chance to match the branching ratio for the semi-leptonic decay $B \to \pi\pi\ell\nu$ with the corresponding decay $B \to \rho\ell\nu$. More precisely we have to integrate the branching ratio over $W \in [m_\rho - \Gamma_\rho, m_\rho + \Gamma_\rho]$, i.e. the ρ-meson pole and compare

$$\left.\frac{\mathrm{d}B(B^\pm \to \nu\ell^\pm\pi^+\pi^-)}{\mathrm{d}q'^2}\right|_{q'^2=0} \equiv \left.\frac{\mathrm{d}B(B^0 \to \nu e^-\rho^+)}{4\mathrm{d}q'^2}\right|_{q'^2=0}. \quad (29)$$

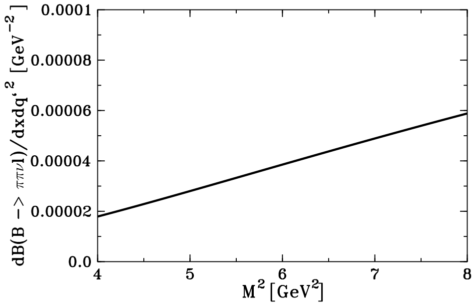

Fig. 3. Borel dependence of the differential branching ratio $B \to \pi\pi\ell\nu$. The value for x is fixed at the ρ pole in the HLS parameterization, i.e. $x = 1 - m_{\rho,\mathrm{HLS}}^2/m_B^2$, with $m_{\rho,\mathrm{HLS}} = 774.57\,\mathrm{MeV}$

Table 1. Sum rule parameters used in the calculation for the estimate of the B-decay width. For the calculation of the decay width the central values of the sum rule parameters have been used, and for the error the maximal deviation to the left and right border

	M^2 [GeV]^2	s_0^B [GeV]^2	m_b [GeV]
left border	4	35	4.7
central value	6	34	4.8
right border	8	33	4.9

Here one factor 1/2 accounts for the fact that we have to consider a ρ^0 wave function instead for a ρ^+ and another factor 1/2 takes into account that we only consider charged pions. $B(\bar{B}^0 \to \rho^+ e^-\bar{\nu})$ at $q'^2 = 0$ only depends on two form factors $A_1(0)$ and $A_2(0)$ [14]:

$$\begin{aligned}
&\left.\frac{\mathrm{d}B(\bar{B}^0 \to \rho^+ e^-\bar{\nu})}{\mathrm{d}q'^2}\right|_{q'^2=0} = \frac{G_F^2|V_{\mathrm{ub}}|^2}{192\pi^3 m_B^3 \Gamma_B}\sqrt{\lambda}H_0^2, \\
&\lambda = (m_B^2 + m_\rho^2)^2 - 4m_B^2 m_\rho^2, \\
&H_0 = \frac{1}{2m_\rho}\Bigg[(m_B^2 - m_\rho^2)(m_B + m_\rho)A_1(0) - \frac{\lambda}{m_B + m_\rho}A_2(0)\Bigg]. \quad (30)
\end{aligned}$$

For the two form factors $A_1(0)$ and $A_2(0)$ we use the values in [14]:

$$\begin{aligned}
A_1(0) &= 0.27 \pm 0.05, \\
A_2(0) &= 0.28 \pm 0.05. \quad (31)
\end{aligned}$$

For the integration of the branching ratio $B(B \to \pi\pi\ell\nu)$ over the ρ pole we take the range $W^2 \in [(m_\rho - \Gamma_\rho)^2, (m_\rho + \Gamma_\rho)^2]$, see Fig. 4. Hereby we use for the variable transformation from x to W^2 the fact that

$$W^2 = P^2 = m_B^2 \frac{1-x}{x}\left(x - \frac{q'^2}{m_B^2}\right) \to m_B^2(1-x). \quad (32)$$

We can now compare the semi-leptonic branching ratio $B \to \pi\pi\ell\nu$ with the equivalent $B \to \rho\ell\nu$, where the

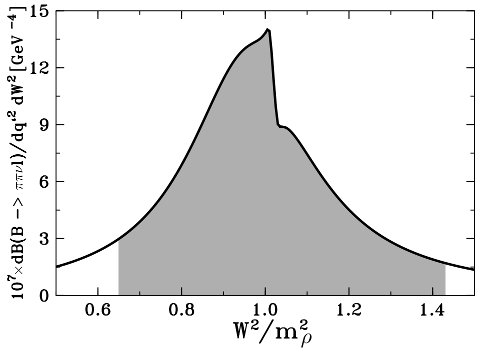

Fig. 4. Integration range over the ρ pole: The shaded area depicts the range of integration in W^2 of the differential branching ratio $B \to \pi\pi\ell\nu$

additional factors are taken into account to make the two quantities comparable. Using the central values of the sum rule parameters for the calculation and obtaining the error from varying the sum rule parameters over the allowed range we obtain

$$\left.\frac{\mathrm{d}B(\bar{B}^0 \to \rho^+\ell^-\bar{\nu})}{4\mathrm{d}q'^2}\right|_{q'^2=0} = \left(13.8^{+28.1}_{-13.8}\right) \times 10^{-7}\mathrm{GeV}^{-2},$$
$$\left.\frac{\mathrm{d}B(B \to \pi\pi\ell\nu)}{\mathrm{d}q'^2}\right|_{q'^2=0} = (3.0 \pm 2.7) \times 10^{-7}\mathrm{GeV}^{-2}. \tag{33}$$

The consistency in the order of magnitude can be taken as a hint that our approach is qualitatively correct. In our case the large error results from the fact that we did neither take into account all 2πDAs nor the higher-twist corrections and restricted ourselves to the asymptotic form only. The big error in case of the $B \to \rho$ semi-leptonic decay comes from the fact that in the kinematic region we consider here, the decay width is the result of two contributions that nearly cancel each other, cf. (30). The number comes from the usual error analysis, and reveals drastic effects. We should state that this is not a numerical estimate of the decay width, but rather a consistency check: we should get the order of magnitude correct. The important task that remains to be done is a modeling of the 2πDAs which will then allow for a quantitative prediction of the semi-leptonic decay of B-mesons into two pions.

5 Summary and conclusions

To summarize, we have shown that the semi-leptonic decay $B^\pm \to \pi^+\pi^-\nu\ell^\pm$ can be described in the LCSR formalism using the two-pion distribution amplitudes. Major observables, except for $f^{\pi\pi}$ and $f_\perp^{\pi\pi}$, are the twist-2 distribution amplitude $g^{\pi\pi}$ and the twist-3 amplitude $e^{\pi\pi}$. When we retain only the twist-2 distribution amplitude $f^{\pi\pi}$, where the asymptotic form is known, and compare the semi-leptonic decay width $B \to \pi\pi\ell\nu$ with the corresponding decay width $B \to \rho\ell\nu$ at $q'^2 = 0$, we find consistency in the order of magnitude.

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A The full formula for the decay width

In the following we give the formula for the decay width $B^+ \to \pi^+\pi^-\ell^+\nu_\ell$ for finite q'^2 and P^2. The electron mass is neglected in the calculation altogether. The formula can be obtained from (18) by the following substitutions:

$$\begin{aligned}
2P\cdot q' &= m_B^2 x + \frac{x-\bar{x}}{x}q'^2,\\
2p_e\cdot q' &= q'^2,\\
2P\cdot p_e &= (m_B^2 x - q'^2)y + \frac{\bar{x}}{x}\frac{p_{e\perp}^2}{y},\\
2R\cdot q' &= \frac{\bar{\zeta}-\zeta}{x}[m_B^2 x^2 + q'^2],\\
2R\cdot p_e &= (\bar{\zeta}-\zeta)\left[(m_B^2 x - q'^2)y - \frac{\bar{x}}{x}\frac{p_{e\perp}^2}{y}\right] + 4\mathbf{K}_\perp\cdot\mathbf{p}_{e\perp}\\
&= (2R\cdot p_e)_\parallel + 4\mathbf{K}_\perp\cdot\mathbf{p}_{e\perp}.
\end{aligned} \tag{A.1}$$

In the limit $q'^2 \to 0$ and $\bar{x} \to 0$ we reproduce the formulas (19). For completeness we add here once more the expressions for R^2, P^2, and $|\mathbf{K}_\perp|^2$:

$$\begin{aligned}
P^2 = W^2 &= m_B^2\frac{1-x}{x}\left(x - \frac{q'^2}{m_B^2}\right),\\
R^2 &= 4m_\pi^2 - W^2,\\
\mathbf{K}_\perp{}^2 &= \zeta\bar{\zeta}W^2 - m_\pi^2.
\end{aligned} \tag{A.2}$$

Using the expressions above we can write down the total decay width, integrated over the polar angle of $\mathbf{p}_{e\perp}$:

$$\begin{aligned}
&\frac{\mathrm{d}\Gamma_B}{\mathrm{d}\zeta\mathrm{d}x\mathrm{d}y\mathrm{d}q'^2\mathrm{d}p_{e\perp}^2}\\
&= \frac{G_\mathrm{F}^2|V_{ub}|^2}{8(4\pi)^5 m_B y^2 x}\frac{\left(1-\frac{q'^2}{m_B^2x^2}\right)}{\left(1+\frac{p_{e\perp}^2}{m_B^2x^2y^2}\right)}\\
&\times\Bigg\{16\Bigg[M_1 P\cdot p_e + M_2(R\cdot p_e)_\parallel\\
&+M_3\left(q'\cdot p_e - \frac{(R\cdot q')(R\cdot p_e)_\parallel}{R^2} - \frac{(P\cdot q')(P\cdot p_e)}{P^2}\right)\Bigg]\\
&\times\Bigg[M_1 P\cdot q' + M_2 R\cdot q'\\
&+M_3\left(q'^2 - \frac{(R\cdot q')^2}{R^2} - \frac{(P\cdot q')^2}{P^2}\right)\Bigg]\\
&-16\Bigg[M_1 P\cdot p_e + M_2(R\cdot p_e)_\parallel
\end{aligned}$$

$$
+M_3\left(q'\cdot p_e - \frac{(R\cdot q')(R\cdot p_e)_\parallel}{R^2} - \frac{(P\cdot q')(P\cdot p_e)}{P^2}\right)\Bigg]^2
$$

$$
-32|\mathbf{K}\perp|^2|\mathbf{p}_{e\perp}|^2\left(M_2 - M_3\frac{R\cdot q'}{R^2}\right)^2
$$

$$
+4M_4^2\frac{R^2}{P^2}\Bigg[-\left(q'^2 - \frac{(R\cdot q')^2}{R^2} - \frac{(P\cdot q')^2}{P^2}\right)
$$

$$
\times\left(\frac{(R\cdot p_e)_\parallel^2}{R^2} + \frac{(P\cdot p_e)^2}{P^2}\right)
$$

$$
-\left(p_e q' - \frac{(R\cdot p_e)_\parallel(R\cdot q')}{R^2} - \frac{(P\cdot p_e)(P\cdot q')}{P^2}\right)^2
$$

$$
-2\frac{|\mathbf{K}_\perp|^2|\mathbf{p}_{e\perp}|^2}{R^2}\left(q'^2 - \frac{(P\cdot q')^2}{P^2}\right)\Bigg]
$$

$$
-4q'^2\Bigg[M_1^2P^2 + M_2^2R^2
$$

$$
+\left(M_3^2 - M_4^2\frac{R^2}{4P^2}\right)\left(q'^2 - \frac{(P\cdot q')^2}{P^2} - \frac{(R\cdot q')^2}{R^2}\right)\Bigg]
$$

$$
-8\frac{M_4}{P^2}\Bigg[\Bigg(P\cdot q'\Big[(P\cdot p_e)(R\cdot q') - (P\cdot q')(R\cdot p_e)_\parallel\Big]
$$

$$
+P^2q'^2\left[(R\cdot p_e)_\parallel - \frac{1}{2}R\cdot q'\right]\Bigg)\left(M_1 - M_3\frac{P\cdot q'}{P^2}\right)
$$

$$
-\Bigg(R\cdot q'\Big[(R\cdot p_e)_\parallel(P\cdot q') - (R\cdot q')(P\cdot p_e)\Big]
$$

$$
+R^2q'^2\left[P\cdot p_e - \frac{1}{2}P\cdot q'\right]\Bigg)\left(M_2 - M_3\frac{R\cdot q'}{R^2}\right)\Bigg]\Bigg\}
$$

$$
\times\frac{M_W^4}{(M_W^2 - q'^2)^2}. \tag{A.3}
$$

In the form given above the decay width is easy to program in a computer code.

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